Grade Eight

The eighth-grade standards are intended to serve two purposes. First, the standards contain content that reviews or extends concepts and skills learned in previous grades. Second, they contain new content that prepares students for more abstract concepts in algebra and geometry. The eighth-grade standards provide students additional instruction and time to acquire the concepts and skills necessary for success in Algebra I. Students will gain proficiency in computation with rational numbers and will use proportions to solve a variety of problems. New concepts include solving multistep equations and inequalities, graphing linear equations, visualizing three-dimensional shapes represented in two-dimensional drawings, and applying transformations to geometric shapes in the coordinate plane. Students will verify and apply the Pythagorean Theorem and represent relations and functions, using tables, graphs, and rules. The eighth-grade standards provide a more solid foundation in Algebra I for those students not ready for Algebra I in grade eight.

While learning mathematics, students will be actively engaged, using concrete materials and appropriate technologies. However, facility in the use of technology shall not be regarded as a substitute for a student's understanding of quantitative concepts and relationships or for proficiency in basic computations. Students will also identify real-life applications of the mathematical principles they are learning that can be applied to science and other disciplines they are studying.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is crucial to a student's understanding and appreciation of the subject. Students should be encouraged to use correctly the concepts, skills, symbols, and vocabulary identified in the following set of standards.

Problem solving has been integrated throughout the six content strands. The development of problem-solving skills should be a major goal of the mathematics program at every grade level. Instruction in the process of problem solving will need to be integrated early and continuously into each student's mathematics education. Students must be helped to develop a wide range of skills and strategies for solving a variety of problem types.

Number and Number Sense

Focus: Relationships within the Real Number System

8.1 The student will
- a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers; and
- b) compare and order decimals, fractions, percents, and numbers written in scientific notation.

8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.

Computation and Estimation

Focus: Practical Applications of Operations with Real Numbers

8.3 The student will
- a) solve practical problems involving rational numbers, percents, ratios, and proportions; and
- b) determine the percent increase or decrease for a given situation.

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables.

8.5 The student will
- a) determine whether a given number is a perfect square; and
- b) find the two consecutive whole numbers between which a square root lies.

Measurement
Focus: Problem Solving

8.6 The student will
- a) verify by measuring and describe the relationships among vertical angles, adjacent angles, supplementary angles, and complementary angles; and
- b) measure angles of less than 360°.

8.7 The student will
- a) investigate and solve practical problems involving volume and surface area of prisms, cylinders, cones, and pyramids; and
- b) describe how changing one measured attribute of a figure affects the volume and surface area.

Geometry
Focus: Problem Solving with 2- and 3-Dimensional Figures

8.8 The student will
- a) apply transformations to plane figures; and
- b) identify applications of transformations.

8.9 The student will construct a three-dimensional model, given the top or bottom, side, and front views.

8.10 The student will
- a) verify the Pythagorean Theorem; and
- b) apply the Pythagorean Theorem.

8.11 The student will solve practical area and perimeter problems involving composite plane figures.

Probability and Statistics
Focus: Statistical Analysis of Graphs and Problem Situations

8.12 The student will determine the probability of independent and dependent events with and without replacement.

8.13 The student will
- a) make comparisons, predictions, and inferences, using information displayed in graphs; and
- b) construct and analyze scatterplots.

Patterns, Functions, and Algebra
Focus: Linear Relationships

8.14 The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship.

8.15 The student will
- a) solve multistep linear equations in one variable with the variable on one and two sides of the equation;
- b) solve two-step linear inequalities and graph the results on a number line; and
- c) identify properties of operations used to solve an equation.

8.16 The student will graph a linear equation in two variables.

8.17 The student will identify the domain, range, independent variable, or dependent variable in a given situation.

GRADE 8

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Transformation of two-dimensional figures

CONNECTION: *Common Core State Standards for Mathematics, Geometry #4 (Grade 8):* Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students manipulate two-dimensional figures based on oral instructions to determine the sequence of transformations of two-dimensional figures in a coordinate plane.

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND congruence of figures in different positions on the coordinate plane.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
LISTENING	Adjust the position of figures based on simple oral commands (e.g., "rotate," "reflect," etc.) using visual supports with a partner	Adjust the position of figures based on oral descriptions (e.g., "reflect over the y-axis") using visual supports with a partner	Adjust the position of figures based on detailed oral descriptions using visual supports with a partner	Adjust the position of figures based on multi-step oral instructions using visual supports	Adjust the position of figures based on information from complex oral discourse	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: geometric transformation, rotation, reflection, translation, dilation, scale factor, vector

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA™

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration will be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to engage in discourse about mathematics with teachers and other students, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop confidence in themselves as mathematics students.

Expressions and Operations

A.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

A.2 The student will perform operations on polynomials, including
- a) applying the laws of exponents to perform operations on expressions;
- b) adding, subtracting, multiplying, and dividing polynomials; and
- c) factoring completely first- and second-degree binomials and trinomials in one or two variables. Graphing calculators will be used as a tool for factoring and for confirming algebraic factorizations.

A.3 The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form.

Equations and Inequalities

A.4 The student will solve multistep linear and quadratic equations in two variables, including
- a) solving literal equations (formulas) for a given variable;
- b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets;
- c) solving quadratic equations algebraically and graphically;
- d) solving multistep linear equations algebraically and graphically;
- e) solving systems of two linear equations in two variables algebraically and graphically; and
- f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.5 The student will solve multistep linear inequalities in two variables, including
- a) solving multistep linear inequalities algebraically and graphically;
- b) justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets;
- c) solving real-world problems involving inequalities; and
- d) solving systems of inequalities.

A.6 The student will graph linear equations and linear inequalities in two variables, including
a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and
b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

Functions

A.7 The student will investigate and analyze function (linear and quadratic) families and their characteristics both algebraically and graphically, including
a) determining whether a relation is a function;
b) domain and range;
c) zeros of a function;
d) *x*- and *y*-intercepts;
e) finding the values of a function for elements in its domain; and
f) making connections between and among multiple representations of functions including concrete, verbal, numeric, graphic, and algebraic.

A.8 The student, given a situation in a real-world context, will analyze a relation to determine whether a direct or inverse variation exists, and represent a direct variation algebraically and graphically and an inverse variation algebraically.

Statistics

A.9 The student, given a set of data, will interpret variation in real-world contexts and calculate and interpret mean absolute deviation, standard deviation, and z-scores.

A.10 The student will compare and contrast multiple univariate data sets, using box-and-whisker plots.

A.11 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. All students are expected to achieve the Geometry standards. The course includes, among other things, properties of geometric figures, trigonometric relationships, and reasoning to justify conclusions. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, algebraic methods, and verbal arguments. A gradual development of formal proof will be encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques, including algebraic skills, should be used to implement these standards. Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), dynamic geometry software, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

Reasoning, Lines, and Transformations

G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- a) identifying the converse, inverse, and contrapositive of a conditional statement;
- b) translating a short verbal argument into symbolic form;
- c) using Venn diagrams to represent set relationships; and
- d) using deductive reasoning.

G.2 The student will use the relationships between angles formed by two lines cut by a transversal to
- a) determine whether two lines are parallel;
- b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs; and
- c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

G.3 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
- a) investigating and using formulas for finding distance, midpoint, and slope;
- b) applying slope to verify and determine whether lines are parallel or perpendicular;
- c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
- d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

G.4 The student will construct and justify the constructions of
- a) a line segment congruent to a given line segment;
- b) the perpendicular bisector of a line segment;
- c) a perpendicular to a given line from a point not on the line;
- d) a perpendicular to a given line at a given point on the line;
- e) the bisector of a given angle,
- f) an angle congruent to a given angle; and
- g) a line parallel to a given line through a point not on the given line.

Triangles

G.5 The student, given information concerning the lengths of sides and/or measures of angles in triangles, will
- a) order the sides by length, given the angle measures;
- b) order the angles by degree measure, given the side lengths;
- c) determine whether a triangle exists; and
- d) determine the range in which the length of the third side must lie.

These concepts will be considered in the context of real-world situations.

G.6 The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

G.7 The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

G.8 The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Polygons and Circles

G.9 The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.

G.10 The student will solve real-world problems involving angles of polygons.

G.11 The student will use angles, arcs, chords, tangents, and secants to
- a) investigate, verify, and apply properties of circles;
- b) solve real-world problems involving properties of circles; and
- c) find arc lengths and areas of sectors in circles.

G.12 The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

Three-Dimensional Figures

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

G.14 The student will use similar geometric objects in two- or three-dimensions to
- a) compare ratios between side lengths, perimeters, areas, and volumes;
- b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
- c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
- d) solve real-world problems about similar geometric objects.

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

Algebra, Functions, and Data Analysis

The following standards outline the content for a one-year course in Algebra, Functions, and Data Analysis. This course is designed for students who have successfully completed the standards for Algebra I. Within the context of mathematical modeling and data analysis, students will study functions and their behaviors, systems of inequalities, probability, experimental design and implementation, and analysis of data. Data will be generated by practical applications arising from science, business, and finance. Students will solve problems that require the formulation of linear, quadratic, exponential, or logarithmic equations or a system of equations.

Through the investigation of mathematical models and interpretation/analysis of data from real life situations, students will strengthen conceptual understandings in mathematics and further develop connections between algebra and statistics. Students should use the language and symbols of mathematics in representations and communication throughout the course.

These standards include a transformational approach to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

The infusion of technology (graphing calculator and/or computer software) in this course will assist in modeling and investigating functions and data analysis.

Algebra and Functions

AFDA.1 The student will investigate and analyze function (linear, quadratic, exponential, and logarithmic) families and their characteristics. Key concepts include
- a) continuity;
- b) local and absolute maxima and minima;
- c) domain and range;
- d) zeros;
- e) intercepts;
- f) intervals in which the function is increasing/decreasing;
- g) end behaviors; and
- h) asymptotes.

AFDA.2 The student will use knowledge of transformations to write an equation, given the graph of a function (linear, quadratic, exponential, and logarithmic).

AFDA.3 The student will collect data and generate an equation for the curve (linear, quadratic, exponential, and logarithmic) of best fit to model real-world problems or applications. Students will use the best fit equation to interpolate function values, make decisions, and justify conclusions with algebraic and/or graphical models.

AFDA.4 The student will transfer between and analyze multiple representations of functions, including algebraic formulas, graphs, tables, and words. Students will select and use appropriate representations for analysis, interpretation, and prediction.

AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Data Analysis

AFDA.6 The student will calculate probabilities. Key concepts include
- a) conditional probability;
- b) dependent and independent events;
- c) addition and multiplication rules;
- d) counting techniques (permutations and combinations); and
- e) Law of Large Numbers.

AFDA.7 The student will analyze the normal distribution. Key concepts include
- a) characteristics of normally distributed data;
- b) percentiles;
- c) normalizing data, using z-scores; and
- d) area under the standard normal curve and probability.

AFDA.8 The student will design and conduct an experiment/survey. Key concepts include
- a) sample size;
- b) sampling technique;
- c) controlling sources of bias and experimental error;
- d) data collection; and
- e) data analysis and reporting.

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. All students preparing for postsecondary and advanced technical studies are expected to achieve the Algebra II standards. A thorough treatment of advanced algebraic concepts will be provided through the study of functions, "families of functions," equations, inequalities, systems of equations and inequalities, polynomials, rational and radical equations, complex numbers, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results should also permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of graphs" from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving and verifying solutions to equations and inequalities. Any other available technology that will enhance student learning should be used.

Expressions and Operations

AII.1 The student, given rational, radical, or polynomial expressions, will
- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
- c) write radical expressions as expressions containing rational exponents and vice versa; and
- d) factor polynomials completely.

AII.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

AII.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.

Equations and Inequalities

AII.4 The student will solve, algebraically and graphically,
- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

Functions

AII.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AII.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include

a) domain and range, including limited and discontinuous domains and ranges;
b) zeros;
c) *x*- and *y*-intercepts;
d) intervals in which a function is increasing or decreasing;
e) asymptotes;
f) end behavior;
g) inverse of a function; and
h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in investigation of functions.

AII.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Statistics

AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AII.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

Expressions and Operations

AII/T.1 The student, given rational, radical, or polynomial expressions, will
- a) add, subtract, multiply, divide, and simplify rational algebraic expressions;
- b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;
- c) write radical expressions as expressions containing rational exponents and vice versa; and
- d) factor polynomials completely.

AII/T.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

AII/T.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of i, and identify field properties that are valid for the complex numbers.

Equations and Inequalities

AII/T.4 The student will solve, algebraically and graphically,
- a) absolute value equations and inequalities;
- b) quadratic equations over the set of complex numbers;
- c) equations containing rational algebraic expressions; and
- d) equations containing radical expressions.

Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII/T.5 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions.

Functions

AII/T.6 The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

AII/T.7 The student will investigate and analyze functions algebraically and graphically. Key concepts include
- a) domain and range, including limited and discontinuous domains and ranges;
- b) zeros;
- c) x- and y-intercepts;
- d) intervals in which a function is increasing or decreasing;
- e) asymptotes;
- f) end behavior;
- g) inverse of a function; and
- h) composition of multiple functions.

Graphing calculators will be used as a tool to assist in the investigation of functions.

AII/T.8 The student will investigate and describe the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph, and factors of a polynomial expression.

Statistics

AII/T.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

AII/T.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

AII/T.11 The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.

AII/T.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

Trigonometry

AII/T.13 The student, given a point other than the origin on the terminal side of an angle, will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of the angle in standard position. Trigonometric functions defined on the unit circle will be related to trigonometric functions defined in right triangles.

AII/T.14 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions, using the definitions and properties of the trigonometric functions.

AII/T.15 The student will find, without the aid of a calculator, the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting angle measures from radians to degrees and vice versa.

AII/T.16 The student will find, with the aid of a calculator, the value of any trigonometric function and inverse trigonometric function.

AII/T.17 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

AII/T.18 The student, given one of the six trigonometric functions in standard form, will
a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, vertical shift, and asymptotes;
c) sketch the graph of the function by using transformations for at least a two-period interval; and
d) investigate the effect of changing the parameters in a trigonometric function on the graph of the function.

AII/T.19 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

AII/T.20 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities.

AII/T.21 The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying solutions to equations and inequalities.

MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

MA.2 The student will apply compositions of functions and inverses of functions to real-world situations. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

MA.3 The student will investigate and describe the continuity of functions, using graphs and algebraic methods.

MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

MA.5 The student will find the sum (sigma notation included) of finite and infinite convergent series, which will lead to an intuitive approach to a limit.

MA.6 The student will use mathematical induction to prove formulas and mathematical statements.

MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

MA.8 The student will investigate and identify the characteristics of conic section equations in (*h, k*) and standard forms. Transformations in the coordinate plane will be used to graph conic sections.

MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and real-world problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations.

MA.10 The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

MA.11 The student will perform operations with vectors in the coordinate plane and solve real-world problems, using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

MA.12 The student will use parametric equations to model and solve application problems.

MA.13 The student will identify, create, and solve real-world problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

MA.14 The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Right triangles

CONNECTION: *Common Core State Standards for Mathematics, Geometry, Similarity, Right Triangles and Trigonometry #6–8 (High School):* Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

EXAMPLE CONTEXT FOR LANGUAGE USE: Students write word problems that can be solved by using right triangles (e.g., finding the height of a tree by using its shadow), and trade with a classmate to solve each other's problems.

COGNITIVE FUNCTION: Students at all levels of English language proficiency CREATE word problems requiring the use of trigonometric ratios and the Pythagorean Theorem to solve.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching
WRITING	Draw and label scenarios for right triangle word problems using illustrated phrase banks	Draw and describe scenarios for right triangle word problems using sentence frames and illustrated phrase banks	Reproduce right triangle word problems using sentence frames and phrase banks	Compose right triangle word problems using textbook models and phrase banks	Compose detailed right triangle word problems using textbook models	

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: sine, cosine, tangent (trigonometric functions), hypotenuse, opposite, adjacent

WIDA

ELD STANDARD 3: The Language of Mathematics

EXAMPLE TOPIC: Mathematical relations & functions

CONNECTION: *Common Core State Standards for Mathematics, Functions, Interpreting Functions #4–6 (Grades 11–12):* For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

EXAMPLE CONTEXT FOR LANGUAGE USE: Students use mathematical abstractions in equations and graphs to represent real-life situations (e.g., using functions and graphs to analyze the lunar cycle, analyze motion graphs of a falling object or parabolic motion).

COGNITIVE FUNCTION: Students at all levels of English language proficiency UNDERSTAND properties of functions.

	Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	
SPEAKING	Name key properties of functions using graphs and equations in L1 or L2 with a partner	Give examples of key properties of functions using labeled graphs and equations with a partner	Describe how key properties of functions are represented using labeled graphs and equations	Summarize representations of key properties of functions in small groups (e.g., think aloud)	Explain with details representations of key properties of functions in small groups	Level 6 – Reaching

TOPIC-RELATED LANGUAGE: Students at all levels of English language proficiency interact with grade-level words and expressions, such as: periodicity, rate of change, quadratic functions, parabola

Figure O: Guiding Questions for the Components of WIDA English Language Development Strands

GRADE:____

WIDA

ELD STANDARD: _____

EXAMPLE TOPIC:
What is one of the topics addressed in the selected content standard(s)?

CONNECTION: Which state content standards, including the Common Core, form the basis of related lessons or a unit of study? What are the essential concepts and skills embedded in the content standards? What is the language associated with these grade-level concepts and skills?

EXAMPLE CONTEXT FOR LANGUAGE USE: What is the purpose of the content work, task, or product? What roles or identities do the students assume? What register is required of the task? What are the genres of text types with which the students are interacting?

COGNITIVE FUNCTION: What is the level of cognitive engagement for the given task? Does the level of cognitive engagement match or exceed that of the content standards?

Language Domain(s):
How will learners process and use language?

Level 1 Entering	Level 2 Emerging	Level 3 Developing	Level 4 Expanding	Level 5 Bridging	Level 6 – Reaching

A Strand of Model Performance Indicators:

What language are the students expected to process or produce at each level of proficiency?

Which language functions reflect the cognitive function at each level of proficiency?

Which instructional supports (sensory, graphic, and interactive) are necessary for students to access content?

TOPIC-RELATED LANGUAGE: With which grade-level words and expressions will all students interact?